Sydney Technical High School

Name: ....

Maths Class: ....

Year 12

Mathematics Advanced

HSC Course

Assessment 1

December, 2019

Time allowed: 70 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A reference sheet is provided.

Section 1 Multiple Choice
Questions 1-7
7 Marks

Section II Questions 8-11
48 Marks

Section I

7 marks

Use Multiple Choice answer sheet for questions 1 – 7

Question 1

What is the value of $6e^{-3}$ correct to 2 decimal places?

(A) 0.28

(B) 0.29

(C) 0.30

(D) 0.31

Question 2

If $y = e^{x^2}$ then $\frac{dy}{dx} =$

(A) $x^2e^{x^2}$

(B) $2xe^{x^2}$

(C) $2e^{x^2}$

(D) $2x^2e^{x^2}$

Question 3

A bag contains 4 red marbles and 6 blue marbles. Three marbles are selected at random without replacement. What is the probability that at least one of the marbles selected is red?

(A) $\frac{1}{6}$

(B) $\frac{1}{2}$

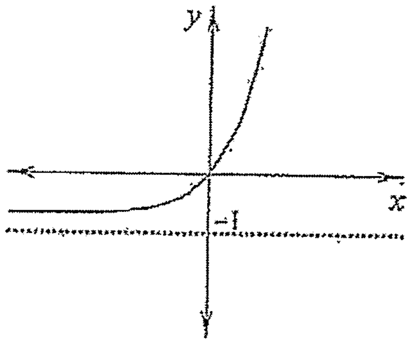
(C) $\frac{5}{6}$

(D) $\frac{29}{30}$

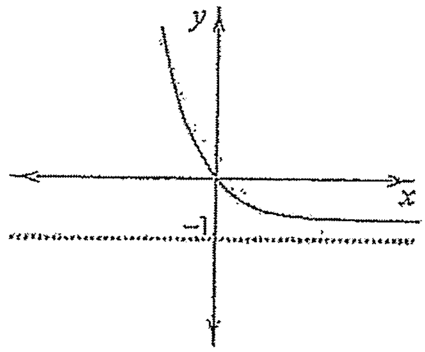
Question 4

What of the following graphs could have the equation $y = 1 - e^{x}$?

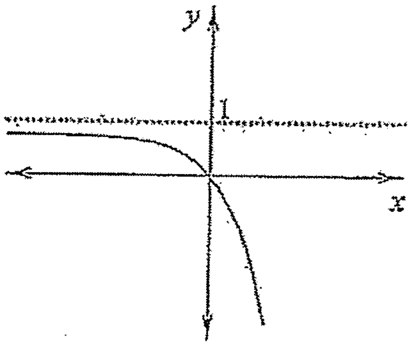
(A)



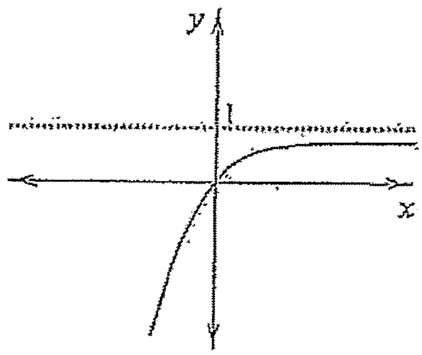
(C)



(B)



(D)



Question 5

Evaluate $\lim_{x \to -3} \frac{x^2-9}{x+3}$

(A) -6

(B) -5

(C) 0

(D) 6

Question 6

Find the value of n given that the following represents a discrete probability distribution.

x	1	2	3	4
$P(X = x)$	$\frac{n}{3}$	$\frac{n}{4}$	$\frac{n}{6}$	n

(A) $\frac{4}{7}$

(B) $\frac{7}{4}$

(C) $\frac{3}{4}$

(D) $\frac{4}{3}$

Question 7

Which expression is equivalent to $4 + log_2 x$?

(A) $log_2(2x)$

(B) $log_2(16 + x)$

(C) $4log_2(2x)$

(D) $log_2(16x)$

End of Section I

Section II

48 marks

Questions 8 – 11

Answer each question in your writing booklet.

Start each question on a NEW sheet of paper

Question 8 **(Start a new page)** **12 marks**

a) Evaluate $log_3 5000$ correct to two decimal places 1

b) Differentiate the following with respect to x

(i) $y = (2 - x)^4$ 1

(ii) $y = 5\sqrt{x}$ 1

(iii) $y = (2x^2 + 4)lnx$ 2

c) Solve $3 = 5e^{-2x}$ 2

d) Find the equation of the tangent to the curve $y = ln(3x)$ at the point where $x = 2$ 3

e) A factory assembles torches. Each torch requires one battery and one bulb. It is known that 6% of all batteries and 4% of all bulbs are defective. 2

Find the probability that, in a torch selected at random, both the battery and the bulb are NOT defective.

End of Question 8

Question 9 **(Start a new page)** **12 marks**

a) Find $f'(x)$, given that $f(x) = (e^x + x)^5$ 2

b) Find $\frac{dy}{dx}$, given that $y = 3^x$ 1

c) $\frac{d}{dx}(log_5 2x)$ 2

d) The total number of boats to be sold next week is shown by the probability distribution below

x	0	1	2	3	4	$\sum$
$p(x)$	0.15	0.30	0.10	0.25	0.20	

Copy the table above into your answer booklet, adding in any necessary rows to complete the following calculations.

(i) Determine the expected value 1

(ii) Find the variance 1

(iii) Find the standard deviation 1

e) Consider the function $f(x) = log_2(x - 2)$

(i) State the domain 1

(ii) Sketch the graph of $y = f(x)$, showing all important features. 2

(Use at least $\frac{1}{3}$ of a page)

(iii) Explain what transformation occurs to $f(x)$ to become $y = -f(x)$ 1

End of Question 9

Question 10 **(Start a new page)** **12 marks**

a) The population of an ant colony can be modelled by using the equation $P = 1000e^{kt}$, where k is a positive constant and t is time in weeks. After two weeks, the population has increased to become 1500.

(i) Show that $k = \frac{1}{2} ln\left(\frac{3}{2}\right)$ 1

(ii) What is the population after four weeks? 1

(iii) After how many weeks will the population exceed 1 million? 2

b) For the probability distribution below, evaluate a and b given that $E(X) = 2.85$ 3

x	1	2	3	4	Σ
$p(x)$	0.1	0.2	a	b	

c) Differentiate $y = ln\left(\frac{3x+1}{2x+2}\right)$ 2

d) If $x = log_{10}(a), y = log_{10}(b)$ and $z = log_{10}(c)$, express the following in terms of x, y and z

(i) $log_{10}\left(\frac{a}{b^2}\right)$ 1

(ii) $log_{10}\left(\sqrt{b^2c}\right)$ 2

End of Question 10

Question 11 **(Start a new page)** **12 marks**

a) Two unbiased six-sided dice are rolled.

(i) What is the probability of obtaining an even sum or a sum greater than 10? 1

(ii) What is the probability that the sum of the numbers is 5, given that one of the dice shows a 2? 1

b) Solve the equation $2log_e x = log_e(5 + 4x)$ 2

c) Tom plays Fifa and has a probability of 0.7 of not winning a game, and a probability of 0.3 of winning.

(i) Find the probability of Tom winning at least one game if he plays three games 2

(ii) What is the least number of consecutive games Tom must play in order to be 90% or more certain, that he will win at least one game? 2

d) The curve $y = ax + \frac{b}{x^2}$ cuts the x axis at the point $(2, 0)$ and the gradient of the normal to this curve at the point $(2, 0)$ equals -1. Find the values of a and b. 4

End of Question 11

End of exam ☺

Year 12 Mathematics Advanced

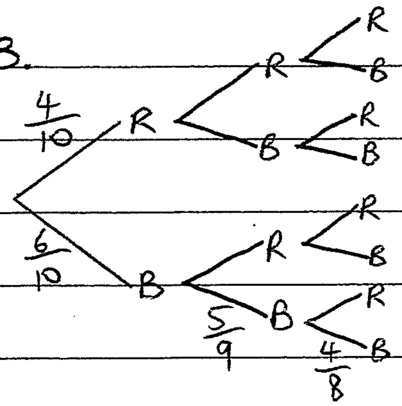
Assessment 1 solutions

1. 0.30 (C)

2. $y=e^{x^2}$

$\frac{dy}{dx}=2xe^{x^2}$ (B)

3.



P(atleast 1 red) = 1 − P(all blue)

$=1-\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}$

$=\frac{5}{6}$ (C)

4. (B)

5. $\lim_{x\to-3}\frac{x^2-9}{x+3}$

$=\lim_{x\to-3}\frac{(x-3)(x+3)}{x+3}$

$=\lim_{x\to-3} x-3$

$=-3-3$

$=-6$ (A)

6. $\frac{n}{3}+\frac{n}{4}+\frac{n}{6}+n=1$

$\frac{7n}{4}=1$

$7n=4$

$n=\frac{4}{7}$ (A)

7. $\log_2 16+\log_2 x$

$=\log_2 2^4+\log_2 x$

$=4\log_2 2+\log_2 x$

$=4+\log_2 x$ (D)

Question 8

a) 7.75 (2 d.p.)

b) (i) $y=(2-x)^4$

$y'=4(2-x)^3\times-1$

$=-4(2-x)^3$

(ii) $y=5\sqrt{x}$

$y=5x^{\frac{1}{2}}$

$y'=\frac{5}{2}x^{-\frac{1}{2}}$ OR $\frac{5}{2\sqrt{x}}$

(iii) $y=(2x^2+4)\ln x$ $\quad u=2x^2+4$ $\quad v=\ln x$

$\frac{du}{dx}=4x$ $\quad \frac{dv}{dx}=\frac{1}{x}$

$y'=4x.\ln x+\frac{2x^2+4}{x}$

8.c) $3=5e^{-2x}$

$\frac{3}{5}=e^{-2x}$

$\ln\left(\frac{3}{5}\right)=\ln e^{-2x}$

$\ln\left(\frac{3}{5}\right)=-2x$

$x=\frac{\ln\left(\frac{3}{5}\right)}{-2}$

$=0.255$ (3 d.p.)

d) $y=\ln(3x)$

when $x=2$, $y=\ln 6$

$\frac{dy}{dx}=\frac{1}{x}$

when $x=2$, $\frac{dy}{dx}=\frac{1}{2}$

$\therefore m_T=\frac{1}{2}$

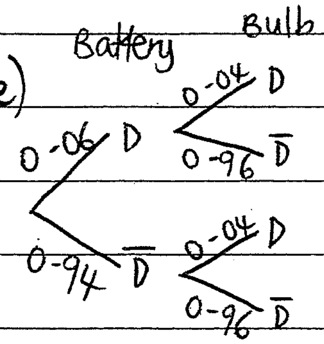
Equation of tangent:

$y-\ln 6=\frac{1}{2}(x-2)$

$y-\ln 6=\frac{1}{2}x-1$

$y=\frac{1}{2}x-1+\ln 6$

e)



P(battery and bulb not defective)

$=0.94\times 0.96$

$=0.9024$

Question 9

a) $f(x)=(e^x+x)^5$

$f'(x)=5(e^x+x)^4\cdot(e^x+1)$

b) $y=3^x$

$\frac{dy}{dx}=\ln 3\cdot 3^x$

c) $\frac{d}{dx}(\log_5 2x)$

$=\frac{d}{dx}\left(\frac{\log_e 2x}{\log_e 5}\right)$

$=\frac{1}{\log_e 5}\cdot\frac{d}{dx}(\log_e 2x)$

$=\frac{1}{\log_e 5}\cdot\frac{2}{2x}$

$=\frac{1}{x\log_e 5}$

d)

x	0	1	2	3	4	Σ
$p(x)$	0.15	0.30	0.10	0.25	0.20	1
$xp(x)$	0	0.30	0.20	0.75	0.80	2.05 — μ
$x^2p(x)$	0	0.30	0.4	2.25	3.2	6.15 — $E(X^2)$

(i) $E(X)=2.05$

(ii) $Var(X)=E(X^2)-\mu^2$

$=6.15-2.05^2$

$=1.9475$

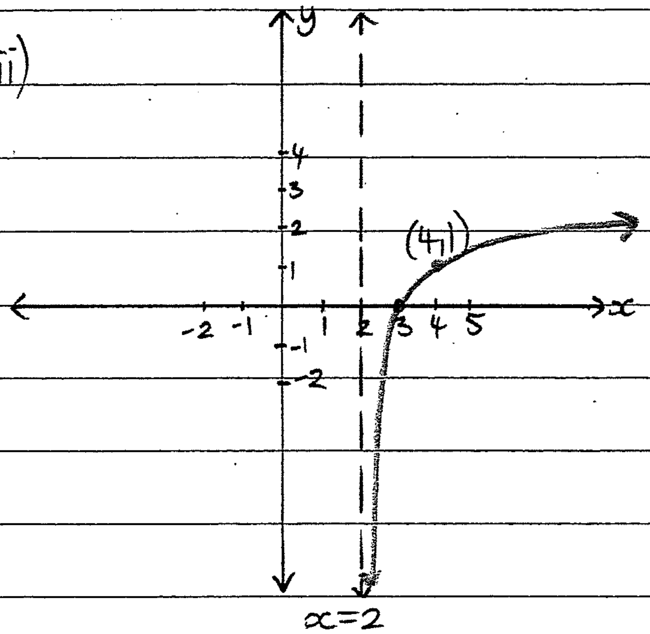
(iii) $\sigma=\sqrt{1.9475}$

$=1.396$ (3 d.p.)

9.e) $f(x) = \log_2(x-2)$

(i) Domain: $x > 2$

(ii)



(iii) Reflected about the x-axis.

Question 10

a)(i) $P = 1000e^{kt}$

$t=2, P=1500$

$1500 = 1000e^{2k}$

$1.5 = e^{2k}$

$\ln 1.5 = \ln e^{2k}$

$\ln 1.5 = 2k$

$k = \frac{\ln 1.5}{2}$

$k = \frac{1}{2}\ln\left(\frac{3}{2}\right)$

(ii) $t=4$, $P = 1000e^{\frac{1}{2}\ln\left(\frac{3}{2}\right)\times 4}$

$= 2250$

$\therefore$ population is 2250 after four weeks

(iii) $1000e^{\frac{1}{2}\ln\left(\frac{3}{2}\right)t} > 1000\,000$

$e^{\frac{1}{2}\ln\left(\frac{3}{2}\right)t} > 1000$

$\frac{1}{2}\ln\left(\frac{3}{2}\right)t > \ln 1000$

$t > \frac{\ln 1000}{\frac{1}{2}\ln\left(\frac{3}{2}\right)}$

$t > 34.07$

$\therefore$ 35 weeks

b) $0.1 + 0.2 + a + b = 1$

$0.3 + a + b = 1$

$a + b = 0.7$ —— ① $\times 3$

$0.1 + 0.4 + 3a + 4b = 2.85$

$3a + 4b = 2.35$ —— ②

$3a + 3b = 2.1$

$-$ $3a + 4b = 2.35$

$-b = -0.25$

$\therefore b = 0.25$

sub into ①

$a = 0.45$

10.c) $y = \ln\left(\frac{3x+1}{2x+2}\right)$

$y = \ln(3x+1) - \ln(2x+2)$

$y' = \frac{3}{3x+1} - \frac{2}{2x+2}$

$= \frac{3(2x+2) - 2(3x+1)}{(3x+1)(2x+2)}$

$= \frac{6x+6-6x-2}{(3x+1)(2x+2)}$

$= \frac{4}{(3x+1)(2x+2)}$

d)(i) $\log_{10}\left(\frac{a}{b^2}\right) = \log_{10}a - \log_{10}b^2$

$= \log_{10}a - 2\log_{10}b$

$= x - 2y$

(ii) $\log_{10}(\sqrt{b^2c}) = \log_{10}\sqrt{b^2} + \log_{10}\sqrt{c}$

$= \log_{10}b + \log_{10}c^{\frac{1}{2}}$

$= \log_{10}b + \frac{1}{2}\log_{10}c$

$= y + \frac{1}{2}z$

Question 11

a)

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5*	6	7	8
3	4	5*	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

(i) P(even or sum > 10) $= \frac{18+2}{36}$

$= \frac{20}{36}$

$= \frac{5}{9}$

(ii) P(sum is 5, given one shows a 2)

$= \frac{2}{11}$

b) $2\log_e x = \log_e(5+4x)$

$\log_e x^2 = \log_e(5+4x)$

$x^2 = 5+4x$

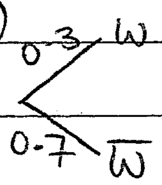
$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5,\ x = -1$

$x = 5$ only as $x > 0$

11\. c)



(i) P(wins atleast one) = 1 − P(loses all)

$= 1 - 0.7^3$

$= 0.657$

(ii) P(wins atleast once in n games) $= 1 - 0.7^n$

$1 - 0.7^n > 0.9$

$0.7^n < 0.1$

$n \log_{10} 0.7 < \log_{10} 0.1$

$n > \dfrac{\log_{10} 0.1}{\log_{10} 0.7}$

$n > 6.45\ldots$

∴ atleast 7 games

d) $y = ax + \dfrac{b}{x^2}$ (2,0)

$0 = 2a + \dfrac{b}{4}$

$0 = 8a + b$ — ①

$y = ax + bx^{-2}$

$y' = a - 2bx^{-3}$

when $x = 2$, $y' = a - 2b(2)^{-3}$

$y' = a - 0.25b$

$m_N = -1$ ∴ $m_T = 1$

$1 = a - 0.25b$ — ②

from ①, $b = -8a$ sub into ②

$1 = a - 0.25(-8a)$

$1 = a + 2a$

$1 = 3a$

$a = \frac{1}{3}$

$b = -8 \times \frac{1}{3}$

$= -\frac{8}{3}$

∴ $a = \frac{1}{3}$, $b = -\frac{8}{3}$

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Question 8:

b) (ii) Change into index form first, then differentiate.

(iii) Most students knew to use the product rule however made errors attempting to simplify the answer. $4xlnx \neq lnx^{4x}$

c) Easier to divide both sides by 5 first and then take the log of both sides. Those who took the log of both sides of the original question incorrectly proceeded. If you are using this method, $ln3 = ln5e^{-2x}$ then becomes $ln3 = ln5 + lne^{-2x}$ using log laws.

d) Pick a form to leave the line in. $\frac{x}{2} - y - 1 + ln6 = 0$ is not general form.
Some students found the derivative of the function, $\frac{1}{x}$, and substituted this into the equation of the tangent as m instead of substituting $x = 2$ in first to get $m = \frac{1}{2}$.

e) You were given the amount of torches that ARE defective. You need to find the probability of torches that are NOT defective first i.e. the complement.

Question 9:

b) Some students failed to recognise this as an exponential and attempted to use the subtracting power method used for polynomials. Students need

c) Many students recognised the change of base and successfully rewrote the expression as $\frac{d}{dx}\left(\frac{\ln 2x}{\ln 5}\right)$, however, several students failed to recognise that $\frac{1}{\ln 5}$ is a constant and then embarked on a futile journey using the quotient rule.

d) This question was generally poorly done. Successful students copied the table and methodically and carefully added rows to obtain E(X), then moved on to the row with $xp(x)$ and $x^2p(x)$. The required formulae for Variance and Expected value are on the formula sheet. Many students did not demonstrate an understanding of how expected value, variance and standard deviation relate to the data obtaining quite obviously incorrect numbers like 65 for standard deviation, where the data only ranges from 0-5

e) A surprising number of students were not able to articulate or show any meaningful understanding of the geometric effect of the basic transformation of $y = -f(x)$

Question 10:

b) Some students did not solve simultaneous equations correctly.

c) Some students did not use the chain rule while they were supposed to do

$$\frac{d}{dx}(\ln(f(x))) = \frac{f'(x)}{f(x)}.$$

Question 11:

a) Students who used grids were more likely to be successful. This made it easier to avoid over-counting. Students should revise conditional probability, many students read "given that one of the dice shows a two" as "AND one of the dice shows a two".

b) Students were fairly successful in forming and solving a quadratic equation, although some did not see relationship $2\log_e x \to \log_e x^2$. Most students left both 5 and -1 as answers, not realising that -1 gives a negative inside a logarithm and should therefore be discarded.

c) This question was fairly well done, with only some silly errors such as swapping 0.3 and 0.7. The second part was often done by trial and error (rather than logarithms), which is a valid technique, but may become impractical for larger numbers.

d) Students are highly advised to set out simultaneous equations clearly, for their marker's benefit as well as their own. Many students tried to find the equation of the normal, and were not able to get correct values of a and b with this method. Instead, they should form two equations using the two pieces of information: At $x = 0, y = 2$, and at $x = 0, y' = 1$.

Students also lost many marks to silly errors, such as $\frac{d}{dx}(ax) = x$ and $\frac{d}{dx}(x^{-2}) = -2x^{-1}$.